

# Computation of the flow over a 2D periodic hill problem with the Discontinuous Galerkin Spectral Element Method

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**Thomas Bolemann, Andrea Beck, Claus-Dieter Munz**

*Institute for Aerodynamics and Gasdynamics, University of Stuttgart, Germany*

*Email: bolemann@iag.uni-stuttgart.de*

## Abstract

In the following work, we present the results of selected simulations of the classical Taylor-Green vortex problem with a variant of the Discontinuous Galerkin method (DG) labeled the “Discontinuous Galerkin Spectral Element Method” (DGSEM). In the classical DGSEM formulation, the non-linear fluxes are co-located on the solution grid, leading to a highly efficient scheme but possible aliasing errors. Polynomial de-aliasing techniques proposed by Kirby and Karniadakis [4] avoid these errors, but incur a higher computational cost. We show results for the co-location and fully de-aliased versions of DGSEM, along with results for a locally adaptive de-aliasing approach.

## The Flow over a Periodic Hill

The flow over a periodic hill is frequently used benchmark case for turbulence modelling, see e.g. [2, 3, 6, 7]. In contrast to the more common square duct hill flows, it covers various physical phenomena like flow separation, reattachment and, depending on the Reynolds number, turbulent transition and relaminarization. Figure 1 shows the instantaneous vortical structures.

The shape of the hill is described in the test case description and in [1], the length of the channel is  $L_x = 9h$  and corresponds to the distance between the two hills with  $h$  denoting the hill height. The channel height is  $L_y = 3.035h$  and the spanwise extent is  $L_z = 4.5h$ . We choose two Reynolds numbers of  $Re_h = 2800$  and  $Re_h = 10595$  based on the bulk velocity  $u_b$  at the hill crest. We use  $Ma = 0.1$  to avoid compressibility effects. We apply isothermal no-slip boundary conditions at the lower and upper walls with a nondimensional temperature of 1, all other boundaries being periodic. We employed the provided  $128 \times 64 \times 64$  mesh, and coarsened it isotropically twice, resulting in 8192 cells for the final mesh. Based on the original mesh, we curved our mesh by agglomeration to achieve a 4<sup>th</sup> order geometry representation. The resulting grid is shown in Figure 2.

## Code Framework

Our code framework is based on a variant of the Discontinuous Galerkin method labeled the “Discontinuous Galerkin Spectral Element method”, see Kopriva [5], and solves the compressible Navier-Stokes equations. The implementation allows the selection of arbitrary polynomial order and thus enables us to study the features of high order formulations very efficiently within our framework. In addition, the integration precision of the flux terms can be chosen independently from the solution to allow polynomial de-aliasing. Explicit time integration is achieved by a 5-stage 4<sup>th</sup> order Runge-Kutta scheme.

The code is accompanied by a postprocessing tool for visualization and a-posteriori extraction of relevant flow features and a 3D Fast Fourier transform for the analysis of flow spectra. The whole framework is fully MPI-parallelized, where special care has been taken to achieve a high parallel efficiency and excellent scaling. On the JuQueen (IBM BlueGene/Q system, Jülich Supercomputing Center) system, a perfect strong scaling was measured on up to 216,000 ranks.

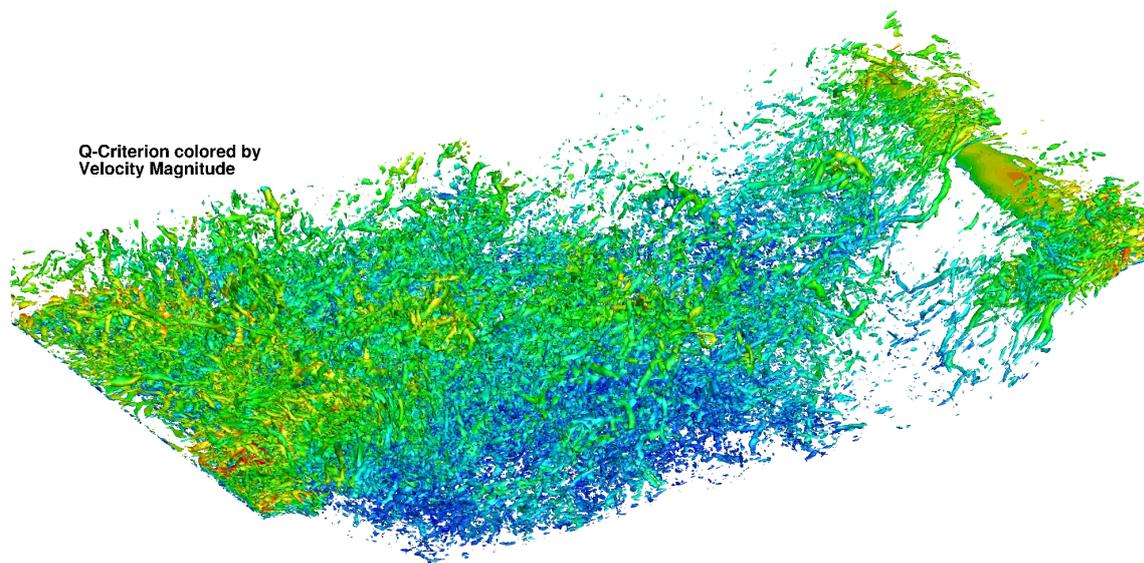


Figure 1: Q-Criterion isocontours evaluated at  $t = 800$ , colored by velocity magnitude.

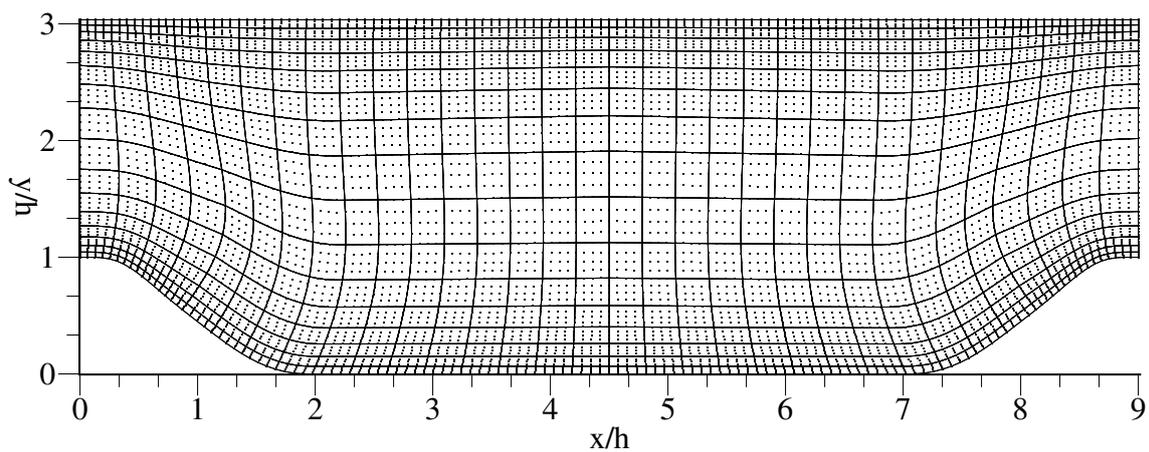


Figure 2: Curved mesh for periodic hill computation.

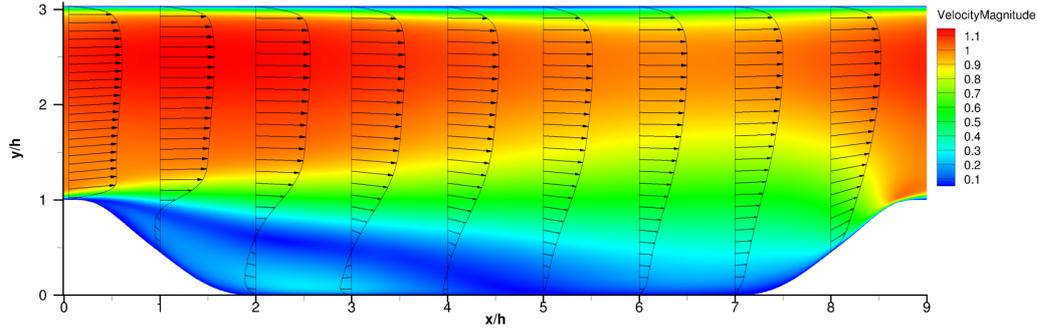


Figure 3: Averaged velocity magnitude and flow profiles at sampling position (position  $x/h = 0.5$  omitted for clarity).

In this work, we present computations performed Cray XE6 “Hermit” cluster (TauBench of 15.1s) at the High Performance Computing Center Stuttgart (HLRS) on up to 6912 cores.

## Results

The following table list the cases considered in this contribution:

Case	Reynolds no.	$N$	DOF	$M$	No. of cores	Wall Time [s]	WU (CPU Time)
1	2800	3	524288				
2	2800	9	8192000				
3	10595	4	1024000				
4	10595	6	2809856				
5	10595	9	8192000				

Table 1: Periodic Hill computations on the mesh with 8192 elements.  $N$ : polynomial degree;  $M$ : polynomial degree of fluxes; WU: Work units on CRAY XE6 “Hermit” with TauBench  $\approx 4.7$ s.

## References

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