High Order Workshop Results for Case 3.3 Taylor-Green Vortex $Re = 1600$

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I. Code Description

A 3D Discontinuous Galerkin (DG) finite element method[1] is used to discretize the compressible Navier-Stokes (CNS) equations. The solver can handle hybrid mixed element meshes (tetrahedra,pyramids,prisms, and hexahedra), curved elements, and incorporates both p-enrichment and h-refinement capabilities using non-conforming elements (hanging nodes). Additional equations that can be solved include a PDE-based artificial viscosity equation and the Spalart-Allmaras turbulence model (negative-SA variant).[2] The implicit solver uses a Newton-Raphson method to solve the non-linear set of equations. These equations are linearized to obtain the full Jacobian. The linear system is solved using a flexible-GMRES[3] (fGMRES) method. To further improve convergence of fGMRES a preconditioner can be applied to the system of equations. Preconditioners that have been implemented include Jacobi relaxation, Gauss-Seidel relaxation, line implicit Jacobi, and ILU(0). The solver is parallelized using MPI.

II. Case Summary

This simulation was solved explicitly using Runge Kutta 4 for 100,000 time steps on 1024 processors. These simulations were performed on the NCAR-Wyoming supercomputer (NWSC) Yellowstone which is a 1.5 Petaflops high performance IBM iDataPlex architecture featuring 72,576 Intel Sandybridge cores (2.6 GHz Intel E5-2670 processors configured in dual socket nodes) and 144.6 TB of memory. The Taubench for this machine is 8.4 seconds.

III. Meshes

A structured cartesian grid is created for this problem. The domain is from $[-\pi, \pi]$ and periodic boundary conditions are used on all boundaries. Two grids are used: one with 32^3 elements and another with 64^3 elements.

IV. Results

Eight simulations varying the basis polynomial degree and grid resolution were performed for the Taylor-Green vortex problem. Polynomial degrees of 1,2,3, and 4 were simulated along with two grids ($n = 32^3$ and $n = 64^3$). Figure 1 plots the Kinetic energy for all eight cases along with the pseudo-spectral kinetic energy. Figure 2 plots the dissipation rate calculated by taking the derivative of kinetic energy with respect to time (using a first order finite difference) along with the pseudo-spectral dissipation rate. Figure 3 plots the dissipation rate calculated using

$$\epsilon_1 = \frac{2\mu}{\rho_0 \Omega} \int_\Omega S^d : S^d \mathrm{d}\Omega$$

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along with the pseudo-spectral dissipation rate. Figure 4 plots the enstrophy along with the pseudo-spectral enstrophy. Figure 5 shows the L_∞ error for the dissipation rate versus work units. Figure 6 shows iso-contours of vorticity magnitude on the $\frac{x}{L} = -\pi$ face at $\frac{t}{t_c} = 8$.

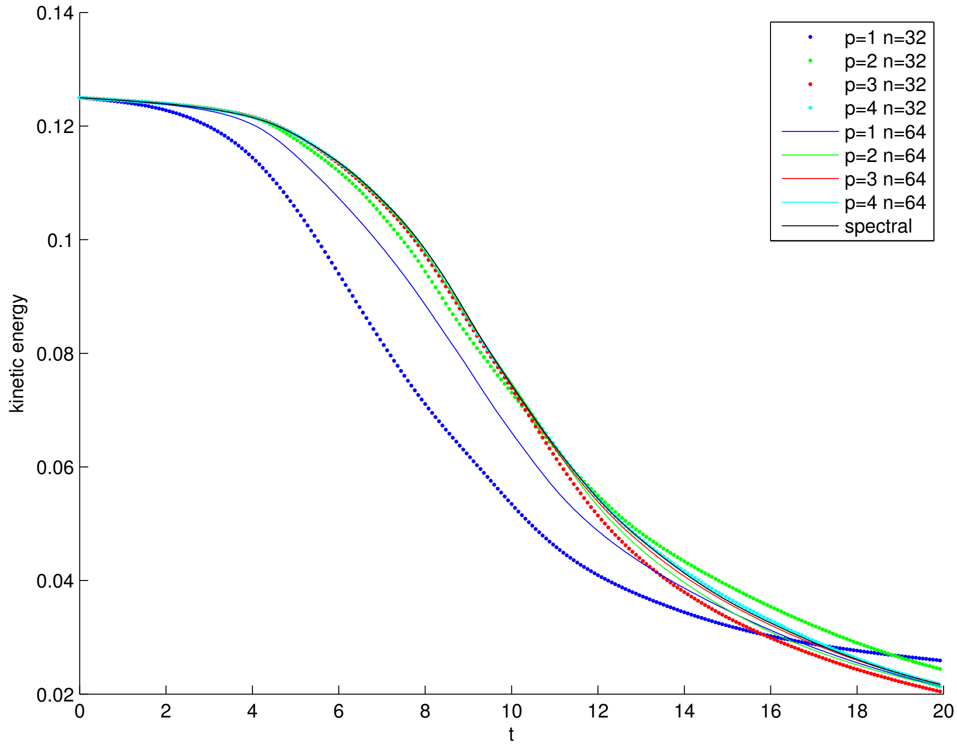

Figure 1. Kinetic energy for the Taylor-Green vortex at $Re = 1600$

References

[1]Brazell, M. J. and Mavriplis, D. J., "3D mixed element discontinuous Galerkin with shock capturing," San Diego, CA, United states, 2013, pp. American Institute of Aeronautics and Astronautics (AIAA) –.

[2]Allmaras, S., Johnson, F., and Spalart, P., "Modifications and Clarifications for the Implementation of the Spalart-Allmaras Turbulence Model," *7th International Conference on Computational Fluid Dynamics*, 2012.

[3]Saad, Y., "A flexible inner-outer preconditioned GMRES algorithm," *SIAM J. Sci. Comput.*, Vol. 14, No. 2, March 1993, pp. 461–469.

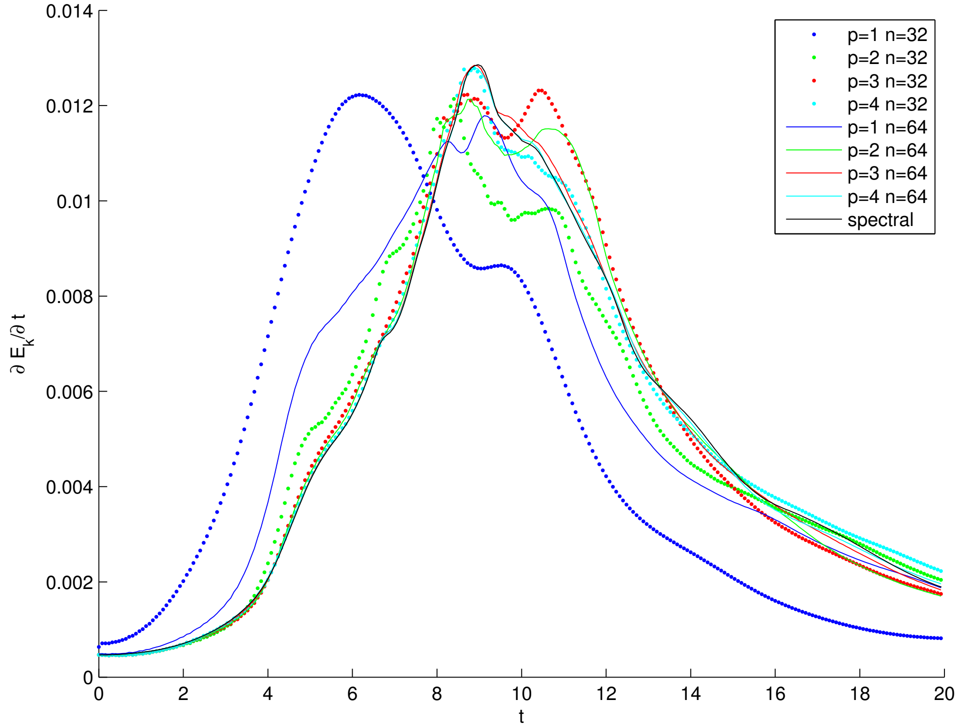

Figure 2. Dissipation rate $\frac{\partial E_k}{\partial t}$ for the Taylor-Green vortex at $Re = 1600$

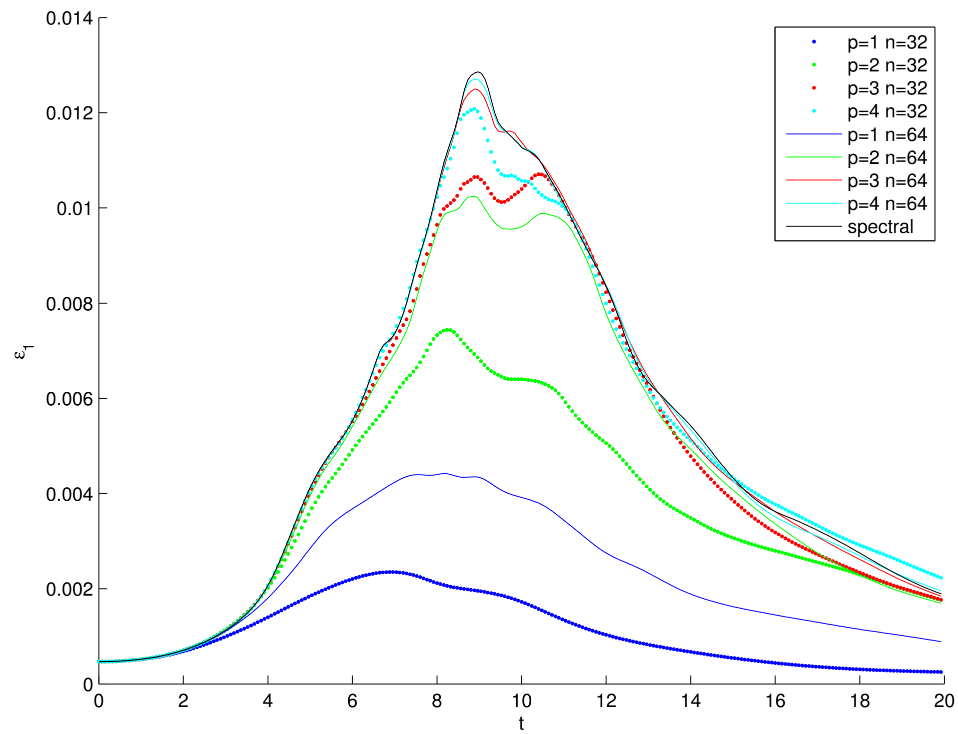

Figure 3. Dissipation rate ϵ_1 for the Taylor-Green vortex at $Re = 1600$

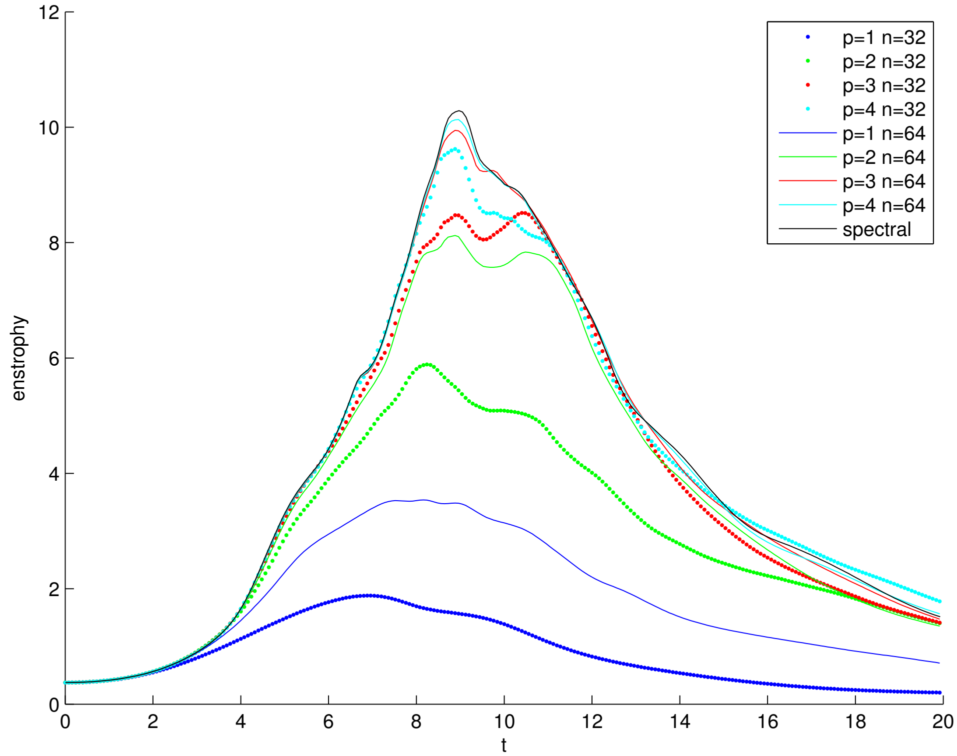

Figure 4. Enstrophy $\mathcal{E}$ for the Taylor-Green vortex at $Re = 1600$

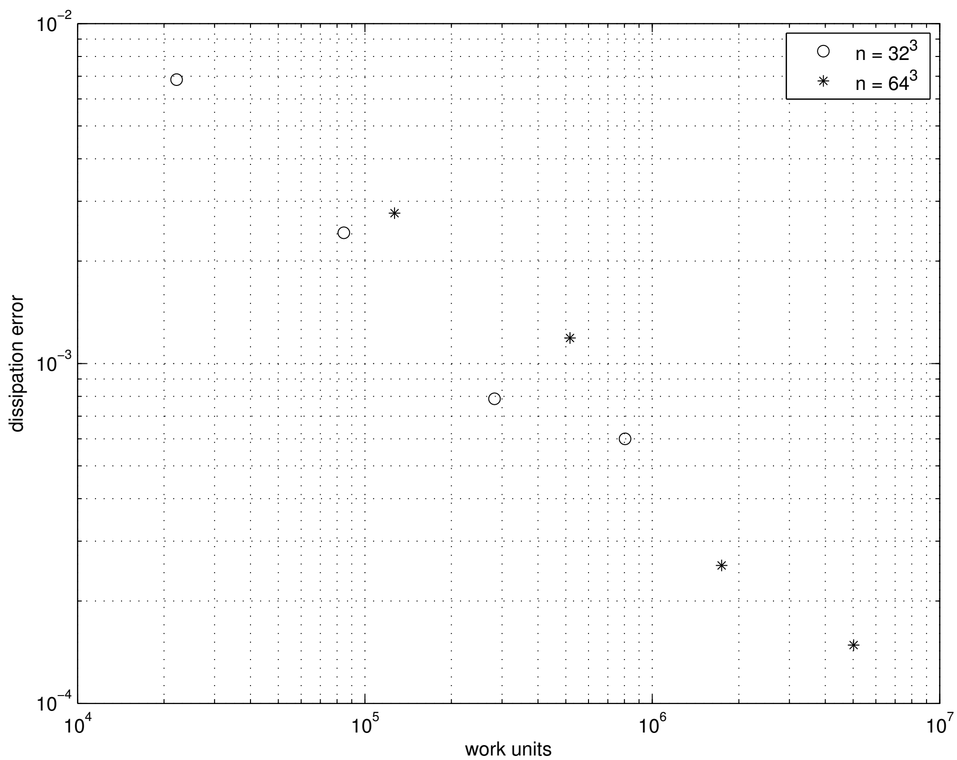

Figure 5. Dissipation error vs work units for the Taylor-Green vortex at $Re = 1600$

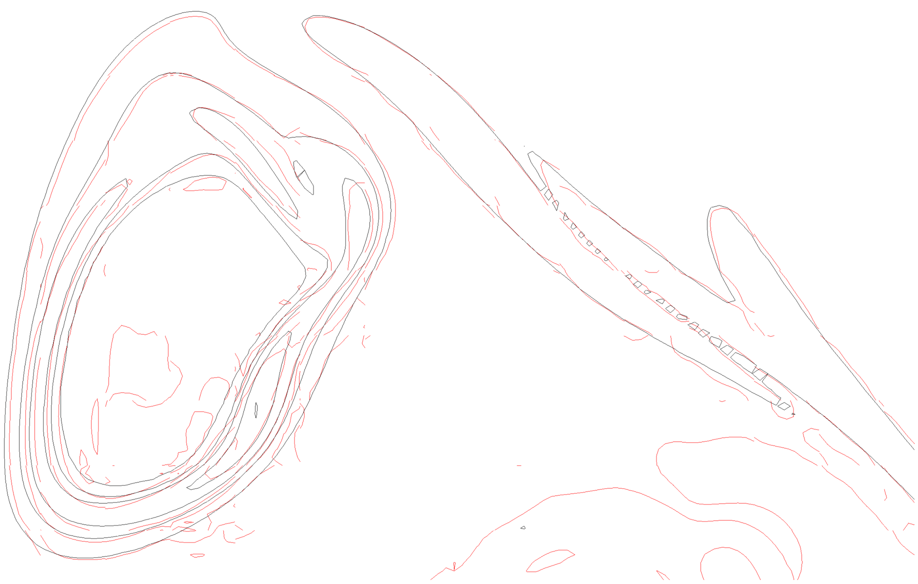

Figure 6. Iso-Contours of vorticity magnitude $\frac{L}{V_0}|\omega| = 15, 10, 20, 30$ at $\frac{t}{t_c} = 8$ and $\frac{x}{L} = -\pi$ for the Taylor-Green vortex at $Re = 1600$, DG $p = 4$, $n = 64^3$ (red), pseudo-spectral (black)