Testcase C 3.1
MDA 30P-30N
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DLR Braunschweig (AS - C²A²S²E)
DG discretization

*Basis functions*

- non-parametric ortho-normal basis functions
- directly formulated in physical space
- also referred to as Taylor-DG
- need to be evaluated for each mesh element

*RANS equations*

- SA turbulence model (negative SA)
- second scheme of Bassi and Rebay (BR2) for viscous terms
- Roe flux as a convective flux, based on an eigen-decomposition of the full jacobian
2D high lift airfoil MDA 30P 30N

- Mach number $M = 0.2$,
- Reynolds number $Re = 9 \cdot 10^6$,
- angle of attack $\alpha = 16^\circ$.

Figure: Pressure for a $p = 2$ solution on 33728 elements mesh.
Testcase 3.1

Mesh hierarchy with own meshes (DLR):

- (structured) quadrilateral meshes with piecewise quartic boundaries
- farfield distance approx. 50 chord lengths
- 2 108, 8 432, 33 728 and 134 912 elements

Figure: Coarsest mesh with 2 108 elements.
Numerical algorithms: Multigrid

- $p$-MG
- $h$-MG based on unstructured agglomeration
Numerical algorithms

*possible solver choices*

- single grid Backward-Euler
- start up strategy in mesh or order sequencing for improved initial conditions
- linear MG as preconditioner
- non-linear MG (FAS) to accelerate process in pseudo-time
- non-linear MG with linear MG on each level
Figure: Convergence of all residual components for an MDA 30P-30N SA-computation with $\rho = 2$ on the 134,912 element mesh.
Figure: Convergence of the density component for an MDA 30P-30N SA-computation with $p = 2$ on the 134,912 element mesh.
Figure: Convergence of the density component for an MDA 30P-30N SA-computation with $p = 2$ on the 134,912 element mesh.
Two graphs show the relationship between DoF$^{-1/2}$ and lift or drag for different polynomial orders ($p=1$, $p=2$, $p=3$). The graphs indicate that as the DoF$^{-1/2}$ decreases, the lift increases, while the drag decreases. The work units for lift and drag are shown on the x-axis and y-axis, respectively.

Legend:
- $p=1$
- $p=2$
- $p=3$
Reference values

Assuming idealized error behavior

\[ C_L = C_{L_{\text{ref}}} + \varepsilon \cdot N^{-\frac{\alpha}{d}} \]

the p=2 results on the finest three meshes have been exploited to obtain

- \( C_{L_{\text{ref}}} = 4.1719 \)
- \( C_{D_{\text{ref}}} = 0.04665 \)
- observed order \( \alpha = 2.5 \)
DoF \sim 10^{-2}

Work units

Drag error

p=1

p=2

p=3

10^0

10^{-1}

10^{-2}

10^{-3}

10^{-4}

10^{-5}

10^2

10^3

10^4

10^5

Lift error