Problem C2.2
Laminar Flow around a Delta Wing

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I. Code Description

`AdHoCfd` is a unified framework for adaptive high-order methods with the main application in Computational Fluid Dynamics. It is written entirely in C++ and makes heavy use of templates to enable a rapid yet efficient way of incorporating new physical models, discretizations, and solvers. In order to build exact implicit operators, fluxes and boundary conditions are automatically differentiated using operator overloading.

Currently, the framework includes a standard and a hybridized discontinuous Galerkin discretization.[1–4] For the latter method, the globally coupled unknowns are only defined on the element interfaces, so that both storage requirements and computational time can be reduced.

The framework is built on top of the finite element package `Netgen/Ngsolve`[5] which provides, among many other things, meshing capabilities, basis functions of arbitrary order, and quadrature rules for a wide range of element types. Results can be visualized with `Tecplot` and `Paraview`.

We use the scientific computing library `PETSc`[6] to solve the arising linear system in implicit discretizations. By default, we apply the generalized minimal residual method (GMRES) preconditioned with an incomplete LU factorization. Besides the matrix orderings already provided by `PETSc` (including the reverse Cuthill-McKee algorithm), the (block) minimum discarded fill (MDF) method[7] is available. As a nonlinear solver we employ a damped Newton method with pseudo-transient continuation. In order to further enhance the robustness of this method, a line search on the residual, physicality checks, and update limiting are available.

Adjoint-based error estimation for various target functionals is readily available to drive *hp*-adaptation (both isotropic and anisotropic[8] in h).

The two and three-dimensional compressible Euler, Navier-Stokes, and RANS equations are available. The latter is complemented with the k-ω turbulence model. All equations are in non-dimensional form.

All computations have been performed in serial on a Mac Pro with 2 quad-core Intel Xeon (2.4 Ghz) and 64 GB of shared memory. One work unit corresponds to 10.43 seconds.

II. Case Summary

In this test case, we consider subsonic, laminar flow around a delta wing. The free stream Mach number is $\mathrm{Ma} = 0.3$, and the Reynolds number based on the mean chord is $\mathrm{Re}_{\mathrm{mc}} = 4000$. The angle of attack is $\alpha = 12.5°$, the ratio of specific heats is $\gamma = 1.4$, and the Prandtl number is given by $\mathrm{Pr} = 0.72$. The viscosity is assumed to be constant.

We use characteristic upwinding at the farfield boundaries. The delta wing is modeled as an isothermal no-slip wall with $T_w = T_\infty$.

We use our primal HDG solver with polynomial degrees ranging from $p = 0$ to $p = 4$ where we initialize a computation with a lower order solution to enhance convergence (see Fig. 1). The CFL number is initialized with 1 and gets amplified by a factor of 100 in every Newton step. We consider a computation to be converged when the l_2 norm of the residual is smaller than 10^{-10}. The linear system is solved to a relative residual tolerance of 10^{-4}. We use 60 Krylov vectors, up to 4 restarted GMRES iterations, and no additional levels of fill in the ILU factorization. We use a Roe-type of stabilization for convection and a BR2-type of stabilization for diffusion.

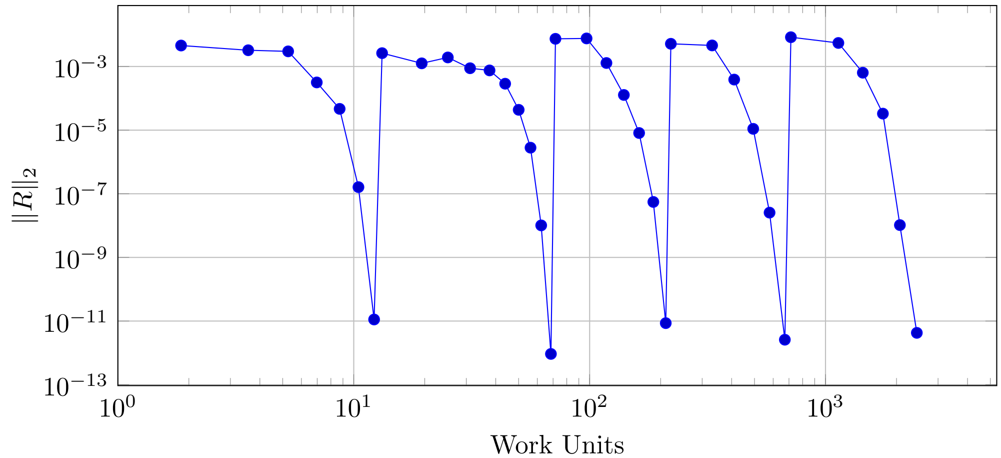


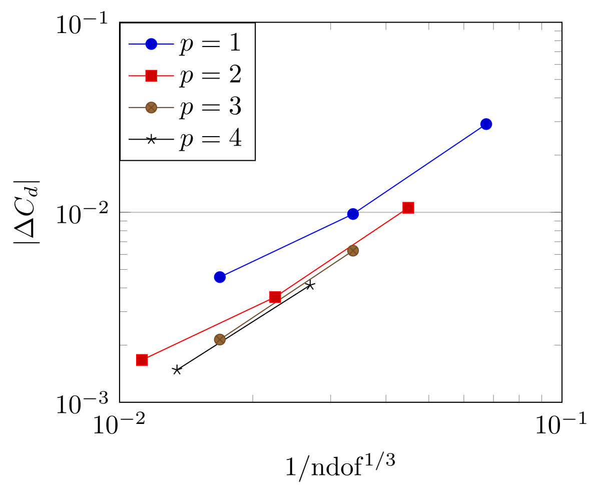
Figure 1: Residual history for $p = 0 \dots 4$ on the 3264-element mesh

III. Meshes

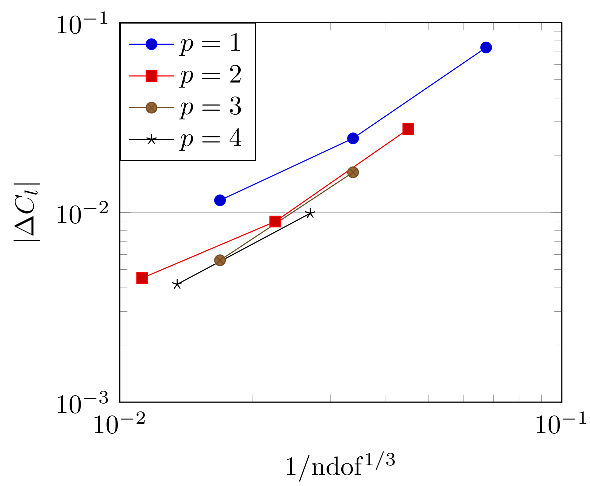
We perform computations on the meshes provided by the workshop (n_e = 408, 3264, and 26112). Please note, that these meshes include degenerated elements (quadrilateral boundary faces with two coinciding vertices and the resulting elements) and zero-measure faces. The first do not pose a problem to our solver. The latter, however, have to be skipped during assembly.

IV. Results

In order to plot the error in lift and drag coefficients, we use the reference values given by Hartmann in the second workshop ($c_{D,\mathrm{ref}} = 0.1658$, $c_{L,\mathrm{ref}} = 0.347$). Due to the sharp edges, convergence in these functionals is suboptimal (between first and second order). Nonetheless, polynomial degrees higher than 1 seem to pay off in accuracy.



(a) Drag coefficient



(b) Lift coefficient

Figure 2: Error with respect to degrees of freedoms

Table 1: Results and timings for varying meshes and polynomial degrees.

(a) $p = 0$

n_e	n_{dof}	C_d	C_l	Work Units
408	408	$5.714600{\cdot}10^{-1}$	$1.159682{\cdot}10^{0}$	$1.295617{\cdot}10^{0}$
3,264	3,264	$4.901848{\cdot}10^{-1}$	$8.326553{\cdot}10^{-1}$	$1.219846{\cdot}10^{1}$
26,112	26,112	$3.735303{\cdot}10^{-1}$	$5.945640{\cdot}10^{-1}$	$1.119394{\cdot}10^{2}$

(b) $p = 1$

n_e	n_{dof}	C_d	C_l	Work Units
408	3,264	$1.949187{\cdot}10^{-1}$	$4.208731{\cdot}10^{-1}$	$6.451793{\cdot}10^{0}$
3,264	26,112	$1.755929{\cdot}10^{-1}$	$3.715385{\cdot}10^{-1}$	$6.854266{\cdot}10^{1}$
26,112	208,896	$1.703596{\cdot}10^{-1}$	$3.585669{\cdot}10^{-1}$	$6.697989{\cdot}10^{2}$

(c) $p = 2$

n_e	n_{dof}	C_d	C_l	Work Units
408	11,016	$1.763615{\cdot}10^{-1}$	$3.744674{\cdot}10^{-1}$	$2.440479{\cdot}10^{1}$
3,264	88,128	$1.693813{\cdot}10^{-1}$	$3.559433{\cdot}10^{-1}$	$2.102643{\cdot}10^{2}$
26,112	705,024	$1.674706{\cdot}10^{-1}$	$3.515042{\cdot}10^{-1}$	$2.082199{\cdot}10^{3}$

(d) $p = 3$

n_e	n_{dof}	C_d	C_l	Work Units
408	26,112	$1.720822{\cdot}10^{-1}$	$3.632558{\cdot}10^{-1}$	$8.560951{\cdot}10^{1}$
3,264	208,896	$1.679395{\cdot}10^{-1}$	$3.525893{\cdot}10^{-1}$	$6.723989{\cdot}10^{2}$

(e) $p = 4$

n_e	n_{dof}	C_d	C_l	Work Units
408	51,000	$1.699363{\cdot}10^{-1}$	$3.568813{\cdot}10^{-1}$	$4.370475{\cdot}10^{2}$
3,264	408,000	$1.672810{\cdot}10^{-1}$	$3.511774{\cdot}10^{-1}$	$2.438825{\cdot}10^{3}$

References

[1]Schütz, J. and May, G., "A hybrid mixed method for the compressible Navier–Stokes equations," *Journal of Computational Physics*, Vol. 240, 2013, pp. 58–75.

[2]Schütz, J. and May, G., "An adjoint consistency analysis for a class of hybrid mixed methods," *IMA Journal of Numerical Analysis*, 2013.

[3]Woopen, M., Balan, A., May, G., and Schütz, J., "A comparison of hybridized and standard DG methods for target-based *hp*-adaptive simulation of compressible flow," *Computers & Fluids*, Vol. 98, 2014, pp. 3–16.

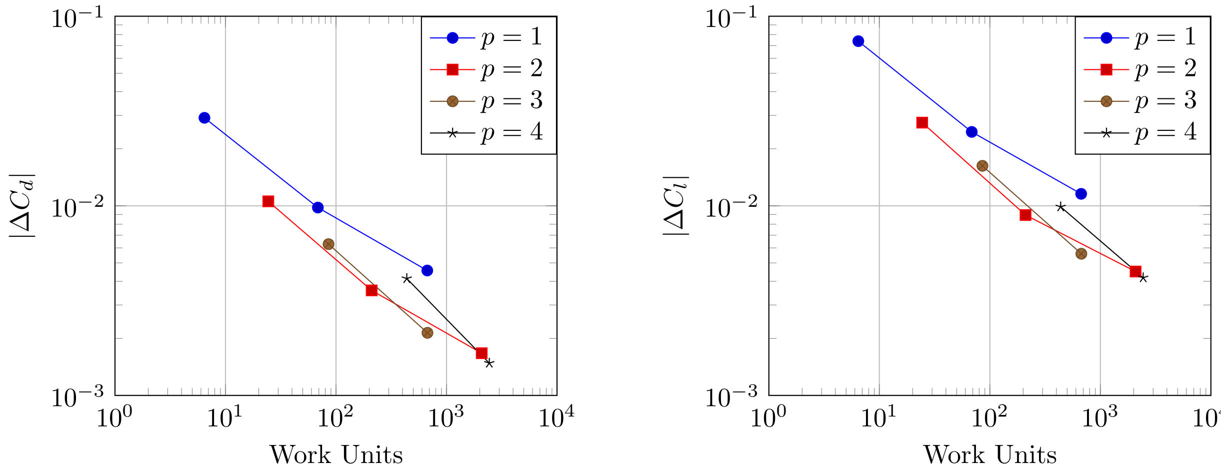
[4]Woopen, M., May, G., and Schütz, J., "Adjoint-based error estimation and mesh adaptation for hybridized discontinuous Galerkin methods," *International Journal for Numerical Methods in Fluids*, 2014.

[5]Schöberl, J., "NETGEN An advancing front 2D/3D-mesh generator based on abstract rules," *Computing and visualization in science*, Vol. 1, No. 1, 1997, pp. 41–52.

[6]Balay, S., Abhyankar, S., Adams, M., Brown, J., Brune, P., Buschelman, K., Eijkhout, V., Gropp, W., Kaushik, D., Knepley, M., McInnes, L. C., Rupp, K., Smith, B., and Zhang, H., "PETSc Web page," .

[7]Persson, P.-O. and Peraire, J., "Newton-GMRES preconditioning for discontinuous Galerkin discretizations of the Navier-Stokes equations," *SIAM Journal on Scientific Computing*, Vol. 30, No. 6, 2008, pp. 2709–2733.

[8]Hecht, F., "BAMG: bidimensional anisotropic mesh generator," *INRIA report*, 1998.



(a) Drag coefficient

(b) Lift coefficient

Figure 3: Error with respect to work units