Problem C1.3
Laminar Boundary Layer on a Flat Plate

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I. Code Description

`AdHoCfd` is a unified framework for adaptive high-order methods with the main application in Computational Fluid Dynamics. It is written entirely in C++ and makes heavy use of templates to enable a rapid yet efficient way of incorporating new physical models, discretizations, and solvers. In order to build exact implicit operators, fluxes and boundary conditions are automatically differentiated using operator overloading.

Currently, the framework includes a standard and a hybridized discontinuous Galerkin discretization.[1–4] For the latter method, the globally coupled unknowns are only defined on the element interfaces, so that both storage requirements and computational time can be reduced.

The framework is built on top of the finite element package `Netgen/Ngsolve`[5] which provides, among many other things, meshing capabilities, basis functions of arbitrary order, and quadrature rules for a wide range of element types. Results can be visualized with `Tecplot` and `Paraview`.

We use the scientific computing library `PETSc`[6] to solve the arising linear system in implicit discretizations. By default, we apply the generalized minimal residual method (GMRES) preconditioned with an incomplete LU factorization. Besides the matrix orderings already provided by `PETSc` (including the reverse Cuthill-McKee algorithm), the (block) minimum discarded fill (MDF) method[7] is available. As a nonlinear solver we employ a damped Newton method with pseudo-transient continuation. In order to further enhance the robustness of this method, a line search on the residual, physicality checks, and update limiting are available.

Adjoint-based error estimation for various target functionals is readily available to drive hp-adaptation (both isotropic and anisotropic[8] in h).

The two and three-dimensional compressible Euler, Navier-Stokes, and RANS equations are available. The latter is complemented with the k-ω turbulence model. All equations are in non-dimensional form.

All computations have been performed in serial on a Mac Pro with 2 quad-core Intel Xeon (2.4 Ghz) and 64 GB of shared memory. One work unit corresponds to 10.43 seconds.

II. Case Summary

In this test case, we consider subsonic, laminar flow along a flat plate. The free stream Mach number is $\mathrm{Ma} = 0.5$, and the Reynolds number based on the length of the plate is $\mathrm{Re}_L = 10^6$. The angle of attack is $\alpha = 0°$, the ratio of specific heats is $\gamma = 1.4$, and the Prandtl number is given by $\mathrm{Pr} = 0.72$. The viscosity is constant.

The plate has length 1, both upstream and top boundary are located at a distance of 0.5 with respect to the plate. We prescribe Riemann invariant in- and outflow boundary conditions on the left, right and top domain boundary, respectively. The bottom boundary ahead of the flat plate is modeled with a symmetry boundary condition. The flat plate itself is an adiabatic no-slip wall.

We use our primal HDG solver with polynomial degrees ranging from $p = 0$ to $p = 4$ where we initialize a computation with a lower order solution to enhance convergence (see Fig. 1). The CFL number is initialized with 100 and gets amplified by a factor of 10 in every Newton step. The linear system is solved to a relative residual tolerance of 10^{-4}. We use 60 Krylov vectors, up to 4 restarted GMRES iterations, and 0 levels of fill in the ILU factorization. We use a Roe-type of stabilization for convection and a BR2-type of stabilization for diffusion.

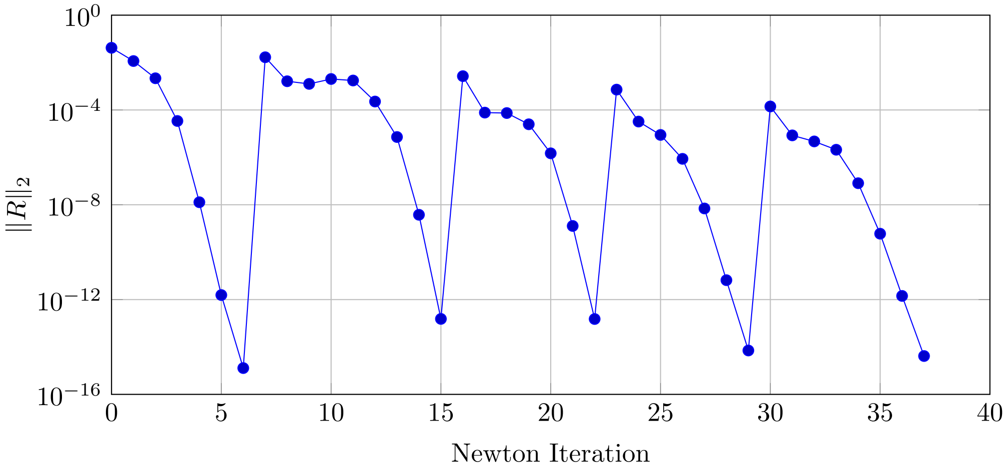

Figure 1: Residual history for $p = 0 \dots 5$ on the coarsest mesh

III. Meshes

The uniform meshes are created by coarsening a fine mesh (taking every other node). The fine mesh has a first element spacing in x- and y-direction of 10^{-6}. There are 288 elements along the plate, 160 elements along the symmetry plane and 320 elements orthogonal to the plate. The coarsest mesh has 9 elements along the plate, 5 elements along the symmetry plane, and 10 elements orthogonal ot the plate. The first element spacing is $\approx 5 \cdot 10^{-5}$.

IV. Results

References

[1]Schütz, J. and May, G., "A hybrid mixed method for the compressible Navier–Stokes equations," *Journal of Computational Physics*, Vol. 240, 2013, pp. 58–75.

[2]Schütz, J. and May, G., "An adjoint consistency analysis for a class of hybrid mixed methods," *IMA Journal of Numerical Analysis*, 2013.

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[4]Woopen, M., May, G., and Schütz, J., "Adjoint-based error estimation and mesh adaptation for hybridized discontinuous Galerkin methods," *International Journal for Numerical Methods in Fluids*, 2014.

[5]Schöberl, J., "NETGEN An advancing front 2D/3D-mesh generator based on abstract rules," *Computing and visualization in science*, Vol. 1, No. 1, 1997, pp. 41–52.

[6]Balay, S., Abhyankar, S., Adams, M., Brown, J., Brune, P., Buschelman, K., Eijkhout, V., Gropp, W., Kaushik, D., Knepley, M., McInnes, L. C., Rupp, K., Smith, B., and Zhang, H., "PETSc Web page," .

[7]Persson, P.-O. and Peraire, J., "Newton-GMRES preconditioning for discontinuous Galerkin discretizations of the Navier-Stokes equations," *SIAM Journal on Scientific Computing*, Vol. 30, No. 6, 2008, pp. 2709–2733.

[8]Hecht, F., "BAMG: bidimensional anisotropic mesh generator," *INRIA report*, 1998.

Table 1: Results and timings for varying meshes and polynomial degrees.

(a) $p = 0$

n_e	n_{dof}	C_d	Work Units
560	560	$3.20766048 \cdot 10^{-3}$	$1.300219 \cdot 10^{-1}$
2,240	2,240	$2.59795598 \cdot 10^{-3}$	$5.098786 \cdot 10^{-1}$
8,960	8,960	$2.25182999 \cdot 10^{-3}$	$2.184265 \cdot 10^{0}$
35,840	35,840	$2.06137508 \cdot 10^{-3}$	$2.543051 \cdot 10^{1}$
143,360	143,360	$1.96024035 \cdot 10^{-3}$	$6.444773 \cdot 10^{1}$

(b) $p = 1$

n_e	n_{dof}	C_d	Work Units
560	2,240	$1.24841379 \cdot 10^{-3}$	$5.583331 \cdot 10^{-1}$
2,240	8,960	$1.30668960 \cdot 10^{-3}$	$2.365189 \cdot 10^{0}$
8,960	35,840	$1.31092965 \cdot 10^{-3}$	$1.098193 \cdot 10^{1}$
35,840	143,360	$1.31122405 \cdot 10^{-3}$	$8.455115 \cdot 10^{1}$
143,360	573,440	$1.31120648 \cdot 10^{-3}$	$4.714006 \cdot 10^{2}$

(c) $p = 2$

n_e	n_{dof}	C_d	Work Units
560	5,040	$1.30886066 \cdot 10^{-3}$	$1.664094 \cdot 10^{0}$
2,240	20,160	$1.31112958 \cdot 10^{-3}$	$5.836529 \cdot 10^{0}$
8,960	80,640	$1.31118267 \cdot 10^{-3}$	$2.650127 \cdot 10^{1}$
35,840	322,560	$1.31118368 \cdot 10^{-3}$	$1.658141 \cdot 10^{2}$
143,360	1,290,240	$1.31118366 \cdot 10^{-3}$	$9.142409 \cdot 10^{2}$

(d) $p = 3$

n_e	n_{dof}	C_d	Work Units
140	2,240	$1.30133356 \cdot 10^{-3}$	$4.702209 \cdot 10^{-1}$
560	8,960	$1.31113143 \cdot 10^{-3}$	$2.953193 \cdot 10^{0}$
2,240	35,840	$1.31118365 \cdot 10^{-3}$	$1.105063 \cdot 10^{1}$
8,960	143,360	$1.31118371 \cdot 10^{-3}$	$4.434264 \cdot 10^{1}$
35,840	573,440	$1.31118367 \cdot 10^{-3}$	$2.637816 \cdot 10^{2}$
143,360	2,293,760	$1.31118352 \cdot 10^{-3}$	$1.415519 \cdot 10^{3}$

(e) $p = 4$

n_e	n_{dof}	C_d	Work Units
140	3,500	$1.30969282 \cdot 10^{-3}$	$9.854328 \cdot 10^{-1}$
560	14,000	$1.31118427 \cdot 10^{-3}$	$5.070994 \cdot 10^{0}$
2,240	56,000	$1.31118375 \cdot 10^{-3}$	$1.767732 \cdot 10^{1}$
8,960	224,000	$1.31118368 \cdot 10^{-3}$	$7.337188 \cdot 10^{1}$
35,840	896,000	$1.31118367 \cdot 10^{-3}$	$4.145389 \cdot 10^{2}$
143,360	3,584,000	$1.31118363 \cdot 10^{-3}$	$2.669256 \cdot 10^{3}$